

ON THE JOINT STATISTICS OF MOBILE RADIO SIGNAL FOR OVERLAPPING CELL AREA IN DIFFERENT FADING CONDITIONS

Leonardo G. de R. Guedes , Michel D. Yacoub and Rodrigo P. Lemos

Universidade Católica de Goiás, Departamento de Computação, Goiânia, Brasil
Universidade Estadual de Campinas, Faculdade de Engenharia Elétrica e de Computação
Universidade Federal de Goiás, Escola de Engenharia Elétrica, Goiânia, Brasil
leonardo.guedes@uol.com.br, admin@icti.com.br, lemos@eee.ufg.br

Abstract. This paper examines the joint statistics of paths to two base stations and estimates the proportion of cell area within which the user's mobile stations are considered to have adequate communication with two or more adjacent base stations in different fading conditions. The calculations are carried out on the instantaneous power basis, where the signal envelope is assumed to follow the Nakagami distribution. It is shown that a substantial proportion of cell area may be considered to have adequate communication with more than one base station in different fading conditions.

1. Introduction

The cells of the mobile radio systems are not clearly defined, but have fuzzy boundaries because of the statistical fluctuation in radio path losses. If the user's mobile stations are near the cell border they may well have adequate communication with more than one base station. The proportion of the cell within which the user's mobile stations are considered to have more than one radio path depends on the fading distribution, on the permissible tolerance in the path loss for satisfactory communication, and on the geographical distribution of cells and mobile stations. This paper estimates the proportion of cell area where the received signals from two adjacent base stations differ by not more than a given tolerance. The calculations are carried out on an instantaneous power basis. It is considered that the received mean signal decreases with distance d from the base station as $d^{-\alpha}$, where α is the path loss coefficient, and that the signal envelope follows the Nakagami distribution.

2. Propagation Model

The mobile radio propagation phenomena are complex and cannot be entirely described by a single model. Many signal strength prediction algorithms are available but their applicability may vary according to the environment. The prediction models

yield the mean signal strength at a given distance from the base station. The input data to such algorithms vary but they all have in common that the mean signal decreases with distance d as $d^{-\alpha}$, where α is the path loss coefficient, typically in the range 2 to 4. However, due to the statistical fluctuations of the various phenomena involving mobile radio propagation, the mobile radio signal cannot be treated only by deterministic methods. Accordingly, several distributions exist that well describe the envelope of the received signal in a mobile environment:

- Lognormal distribution due to shadowing;
- Rayleigh distribution due to multipath propagation;
- Rician distribution due to multipath propagation with a line-of-sight component;
- Suzuki distribution, that combines shadowing and multipath propagation;
- Nakagami distribution, which is a more general distribution that can be used to better characterize the rapid fading.

In this paper we explore the Nakagami case. The other cases are explored in [2-6].

2.1. Nakagami Distribution

The Nakagami distribution was introduced in the early 1940's to characterize the rapid fading in long distance high frequency propagation [1]. In essence, the Nakagami distribution is a central chi-square distribution defined as [1,5,6]

$$M(r, m, \Omega) = p(r) = \frac{2m^m r^{2m-1}}{\Gamma(m)\Omega^m} \exp\left(-\frac{mr^2}{\Omega}\right) \quad (1)$$

where $m = \Omega^2 / \text{Var}(r^2) \geq 1/2$ is the fading factor, $\Omega = E[r^2]$, r is the signal envelope and [5,6]

$$\bar{r} = E[r] = \frac{\Gamma(m + 1/2)}{\Gamma(m)} \sqrt{\frac{\Omega}{m}} \quad (2)$$

The rationale for using the Nakagami distribution to model multipath fading is based on the following two characteristics of the Nakagami distribution:

1. the Rayleigh distribution is a special case of the Nakagami distribution when $m = 1$;
2. the Nakagami distribution can closely approximate the Rician distribution by using the relation between the Rice factor k and m as given by 3.

$$k = \frac{\sqrt{m^2 - m}}{m - \sqrt{m^2 - m}}, \quad m > 1 \quad (3)$$

Power Density Function:

Let $w = r^2/2$ be the received signal power and $\bar{w} = E[w]$. Then, by using a transformation of random variables, the probability density function $p(w)$ of w of a Nakagami faded signal is

$$p(w) = \left(\frac{m}{\bar{w}}\right)^m \frac{w^{m-1}}{\Gamma(m)} \exp\left(-\frac{mw}{\bar{w}}\right) \quad (4)$$

2.2. Distribution of the Ratio of Two Nakagami Variates

Let r_1 and r_2 be two independent Nakagami variables, with parameters m_1, Ω_1 and m_2, Ω_2 , respectively. Then, the distribution $p(F)$ of the ratio $F = (r_1^2/\Omega_1)/(r_2^2/\Omega_2)$ is found to be [1,5-6]

$$p(F) = \frac{\Gamma(m_1 + m_2)}{\Gamma(m_1)\Gamma(m_2)} \left(\frac{m_1}{m_2}\right)^{m_1} F^{m_1-1} \left(1 + \frac{m_1}{m_2} F\right)^{-(m_1+m_2)} \quad (5)$$

The ratio between Ω_1 and Ω_2 , is given by

$$b = \frac{\Omega_2}{\Omega_1} = \frac{E[r_2^2]}{E[r_1^2]} = \frac{\bar{w}_2}{\bar{w}_1} \quad (6)$$

3. The Geographical Distribution of Instantaneous Power Ratio

The occurrence of fading will modify the distribution of the signal strength ratio. Not all user's mobile stations within the border zone will have paths within the assigned tolerance, because one or both path losses may depart from the mean value. On the other hand, some user's mobile stations outside the border zone will have such pair of paths. It is possible to estimate the overall proportion of the cell area with satisfactory path-pair on an instantaneous rather than a mean criterion. Refer to Figure 1, where x is a position variable. Let the density of user's mobile stations in the vicinity of x be $d(x)$. Let $P(a, x)$ be the probability that a mobile, having a location described by the position variable x , have a path-pair within the tolerance $A = 20 \log a$ dB. It is also required that neither of the two signals be bellow a given threshold level c in order to account for coverage requirement. However, according to Reference 2 where the Lognormal and Rayleigh cases are explored, the error committed when c is considered to be zero is very small as compared to the true values obtained for likely values of c . The unconditional probability, then

$$\gamma = \int_0^1 d(x)P(a, x)dx \quad (7)$$

is the mean proportion of user's mobile stations within a cell having a path-pair within tolerance allowing for fading. In an hexagonal array the density $d(x) = 2(1-x)$ gives a good approximation to the distribution of a uniform cell [2]. The probability $P(a, b)$, given the mean power ratio b , can be determined by estimating the probability that the instantaneous signal ratio r_1/r_2 be within the range $1/a$ and a , i.e.,

$$P(a, b) = \text{prob}\left(\frac{1}{a} \leq \frac{r_1}{r_2} \leq a\right) \quad (8)$$

$$P(a, b) = \text{prob}\left[\left(\frac{1}{a}\right)^2 \frac{\Omega_2}{\Omega_1} \leq F \leq a^2 \frac{\Omega_2}{\Omega_1}\right] \quad (9)$$

Suppose that the mean signal decreases with distance x as $x^{-\alpha}$, where α is the path loss slope. Consider a mobile at distances x_1 and x_2 from two base stations of equal power. The ratio \bar{w}_2/\bar{w}_1 of the received signal is

$$b = \left(\frac{\bar{w}_2}{\bar{w}_1}\right) = \left(\frac{x_2}{x_1}\right)^\alpha \quad (10)$$

Clearly, $b = \bar{w}_2/\bar{w}_1 = \Omega_2/\Omega_1$ is a function of the distance x . In particular, considering Figure 1, from (6), (9), (10) and (11):

$$b = \left(\frac{\Omega_2}{\Omega_1}\right) = \left(\frac{x_1}{x_2}\right)^\alpha \cong \left(\frac{1+x}{1-x}\right)^\alpha \quad (11)$$

$$P(a, b) = \int_{F_1}^{F_2} p(F) dF = \int_{\left(\frac{1}{a}\right)^2 \frac{\Omega_2}{\Omega_1}}^{a^2 \frac{\Omega_2}{\Omega_1}} J.F^{m_1-1} \left(1 + \frac{m_1}{m_2} F\right)^{-(m_1+m_2)} dF \quad (12)$$

$$J = \frac{\Gamma(m_1 + m_2)}{\Gamma(m_1)\Gamma(m_2)} \left(\frac{m_1}{m_2}\right)^{m_1} \quad (13)$$

stations are considered to have more than one radio path within the assigned tolerance. The integral in (16) does not appear to lend itself to a closed-form evaluation, but has been evaluated numerically [5,6]. The integrals in (7) and (12) have also been evaluated numerically for non integer values of $m_1 \neq m_2$.

4. Results

We have estimated the proportion of cell area where the received signals from the adjacent base stations differ by not more than a given tolerance. It was considered that the user's mobile stations are within a cell with factor m_2 and all the adjacent cells have the same factor m_1 . In Figure 2 we have plotted the proportion γ of the cell area within which the user's mobile stations have a path-pair within tolerance of A dB for several values of the fading factor m_2 . We have considered that the signal envelope received from all adjacent cells is Nakagami distributed with $m_1 = 1$, which is equivalent to the Rayleigh fading condition. If we assume a tolerance of $A = 8$ dB as a satisfactory condition for adequate communication, the proportion γ is in the range 30% - 50%. In Figure 3 we have plotted the proportion γ for several values of the fading factor m considering that the signal envelope received from all cells is Nakagami distributed with the same fading factor m . In this case γ gives the proportion of the cell area within which the user's mobile stations are considered to have more than one radio path within the assigned tolerance. For a given tolerance of $A = 8$ dB this proportion is in the range 35% - 45%. In Figure 4 we have plotted the proportion γ for different values of the path loss coefficient α considering that the signal envelope received from all cells is Nakagami distributed with the same fading factor $m = 4$. For a given tolerance of $A = 8$ dB this proportion is in the range 37% - 47%.

5. Conclusion

We have estimated the proportion of a cell area within which user's mobile stations are considered to have adequate communication with more than one base station. The calculations were carried out on an instantaneous power basis considering two adjacent base stations in different fading conditions. It can be shown that, for a given path loss slope and a given tolerance, such a proportion is greatly dependent on the fading conditions, therefore, on the statistics of the received envelope. A more useful result can be obtained if the appropriate distribution of the user's mobile stations within the cell is considered to be known. Overall it can be concluded that for a given tolerance the proportion of the cell area where user's mobile stations may have access to more than one base station is rather substantial. For instance, if it is accepted that paths within 8 dB from each other are valid alternatives, such a proportion is in the range 30% - 50%. In fact, tolerances higher than this must be allowed in order to

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avoid unnecessary hand-offs. The study of the proportion of overlapped cell area can be used in cell planning, considering the traffic and hand-off aspects.

6. References

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